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Column buckling of structural bamboo

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Abstract

Bamboo scaffolding is widely used in construction in the South East Asia, in particular, the Southern China and Hong Kong for many decades. However, bamboo scaffolds are generally erected by scaffolding practitioners based on their intuition and experiences without any structural design. In general, column buckling is considered to be one of the critical modes of failure in bamboo scaffolds, often leading to their overall collapse.

This paper presents a research and development project for structural bamboo where the column buckling behaviour of two structural bamboo species, namely *Bambusa pervariabilis* (or Kao Jue) and *Phyllostachys pubescens* (or Mao Jue) were investigated. A total of 72 column buckling tests with bamboo culms of typical dimensions and properties were executed to study the column buckling behaviour of structural bamboo. Furthermore, a limit state design method against column buckling of structural bamboo based on modified slenderness was established and carefully calibrated against test data. It is shown that for Kao Jue, the average model factors of the proposed design method are 1.63 and 1.86 for natural and wet conditions, respectively. Similarly, the average model factors of the proposed design method for Mao Jue are 1.48 and 1.67 for natural and wet conditions, respectively. Consequently, the proposed design method is shown to be adequate.

With the availability of design data on the dimensions and the mechanical properties of structural bamboo together with the proposed column buckling design rule, structural engineers are encouraged to take the advantage offered by bamboo to build light and strong bamboo structures to achieve enhanced economy and buildability.

Keywords: Bamboo scaffolds; Green construction; Structural bamboo; Bamboo columns; Column buckling tests

1. Introduction

Timber is regarded as a good natural construction material, and probably, one of the oldest known materials used in construction. In a modern structural timber code [1], ultimate limit state design philosophy is adopted and structural adequacy is assessed with characteristic values of both loading and resistance using appropriate partial safety factors. Bamboo is another natural construction material and there are over 1500 different botanical species of bamboo across the globe. In general, it is considered that the mechanical properties of bamboo are likely to be at least similar, if not superior, to those of structural timber. In many countries, bamboo culms of various species have been used traditionally as structural members in both temporary and permanent structures under light loads, such as low-rise houses, short span footbridges, roof structures of medium span and assess scaffolds. Furthermore, as bamboo grows very fast and usually takes 3–6 years to harvest, depending on the species and the plantation, there is a growing global interest in developing bamboo as a substitute for structural timber in construction. The effective use of structural bamboo will mitigate the pressures on the ever-shrinking natural forests in developing countries, and thus, facilitate the conservation of the global environment.

However, a major constraint to the development of structural bamboo as a modern construction material is the lack of design standards on both mechanical properties and structural adequacy. As natural non-homogenous organic materials, large variations of physical properties along the length of bamboo members are apparent: external and internal diameters, dry density

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Nomenclature

D_e, D_i	external and internal diameters, respectively
$E_{c,d}$, $E_{b,d}$	design Young's modulus against compression and bending, respectively
$f_{c,k}$, $f_{c,d}$	characteristic and design compressive strengths, respectively
$f_{b,k}$, $f_{b,d}$	characteristic and design bending strengths, respectively
f_{cr}	elastic critical buckling strength
$f_{cc,d}$, $f_{cc,t}$	design and measured compressive buckling strengths, respectively
$\bar{\lambda}$	modified slenderness ratio defined as $\sqrt{\frac{f_{c,d}}{f_{cr}}}$
$\bar{\psi}_c$, $\bar{\psi}_t$	design and measured strength reduction factors for compressive buckling defined as $\frac{f_{cc,d}}{f_{c,d}}$ and $\frac{f_{cc,t}}{f_{c,d}}$, respectively with γ_m equal to 1.0
γ_m	a partial safety factor for material strength

and moisture content. While structural engineers also expect variations in the mechanical properties of structural bamboo, they tend to accept that the mechanical properties of bamboo are likely to be more consistent than those of concrete.

This paper presents a research and development project for structural bamboo where the column buckling behaviour of two structural bamboo species, namely *Bambusa pervariabilis* (or Kao Jue) and *Phyllostachys pubescens* (or Mao Jue) were investigated. A total of 72 column buckling tests with bamboo culms of typical dimensions and properties were executed to study the column buckling behaviour of structural bamboo. A limit state design method against column buckling of structural bamboo based on modified slenderness was then established for general design after careful calibration against test data.

1.1. Bamboo scaffolds

Bamboo scaffolds have been widely used in construction in the South East Asia, in particular, the Southern China and Hong Kong for many decades. Figs. 1 and 2 illustrate typical applications of bamboo scaffolds in Hong Kong [2] as single layered bamboo scaffolds (SLBS) and double layered bamboo scaffolds (DLBS), respectively. In spite of open competition with many metal scaffolding systems imported from countries all over the world, bamboo scaffold remains one of the most preferred access scaffolding systems in building construction in Hong Kong and the neighbouring areas. Both Kao Jue and Mao Jue are commonly used in access scaffolds in Hong Kong, and typical dimensions of both Kao Jue and Mao Jue are presented in Fig. 3. At present, the typical height of bamboo scaffolds is 15 m and the installation of steel bracket supports at regular intervals allow full coverage of building height.

It should be noted that bamboo scaffolds are generally erected by scaffolding practitioners based on their intuition and experiences without any structural design. In general, column buckling is considered to be one of the critical modes of failure in bamboo scaffolds, often leading to their overall collapse. In order to ensure structural adequacy of bamboo scaffolds and other slender bamboo structures in building construction, it is highly desirable to provide rational design rules against column buckling of structural bamboo. With a suitable choice of partial safety factors, structural engineers are thus able to design bamboo structures at a known level of confidence against column buckling.

Fig. 1. Single Layered Bamboo Scaffolds (SLBS).

Fig. 2. Double Layered Bamboo Scaffolds (DLBS).

1.2. Recent research in structural bamboo

Structural bamboo have been used traditionally in China, Philippines, India and Latin America for many hundreds of years, but little research was reported in the past. Recent scientific investigations on bamboo as construction materials were reported by Au et al. [3] in Hong Kong and also by Janssen [4] in Holland. A large amount of data of the mechanical properties for various bamboo species all over the world was also reported by Janssen [5]. While these sets of data provide typical values of compressive, bending and shear strengths of various bamboo species, no characteristic strengths for modern structural design were provided.

A series of experimental studies on structural bamboo were reported by Arce-Villalobos [6] and practical connection details for bamboo trusses and frames were also proposed and tested. Moreover, a recent study on the traditional design and construction of bamboo in low-rise housing in Latin America was conducted and reported by Gutierrez [7]. It is interesting to note that bamboo was classified by Amada et al. [8] as a smart natural composite material with optimized distribution of fibers and matrices, both across cross sections and along member lengths, in resisting environmental loads in nature.

In order to promote the effective use of structural bamboo in building construction, it is essential to pro-

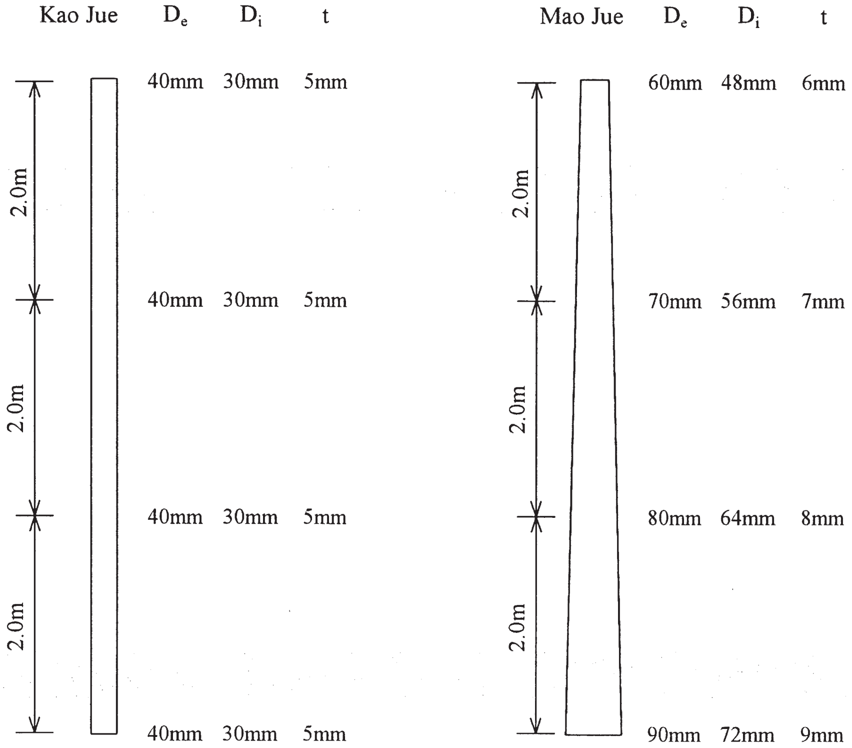


Fig. 3. Typical dimensions of Kao Jue and Mao Jue.

vide basic design data of mechanical properties and design rules against various modes of failure in accordance with modern design philosophy. A pilot study was carried out by Chung and Yu [9] to examine the variation of compressive strength against various physical properties along the length of bamboo culms for both Kao Jue and Mao Jue. Moreover, systematic test series with a large number of compression and bending tests were also executed [10,11] to establish characteristic values of both the strengths and the Young's moduli of each bamboo species for limit state structural design. As shown in Table 1, both Kao Jue and Mao Jue are good constructional materials with excellent mechanical properties against both compression and bending.

1.3. Buckling loads for non-prismatic tubular columns

In general, many researchers have studied the buckling behaviour of non-prismatic columns such as non-prismatic columns of wide flange I-sections, box sections, and solid sections with different support conditions. Gere and Carter [12] presented both exact and approximate solutions for the critical buckling loads of non-prismatic columns, but no solutions for non-prismatic tubular columns were given. Furthermore, Fogel and Ketter [13] examined the elastic buckling loads of a simply supported column with tapered rectangular cross-section of constant thickness under combined compression and bending. The study was then extended to similar columns with different support conditions.

For non-prismatic columns with common solid and tubular cross sections with different support conditions, Williams and Aston [14] presented approximate design curves to assess the lower bounds of elastic buckling loads. However, no closed-formed analytical solutions were provided. More recently, Arce-Villalobos evaluated the critical buckling loads of bamboo columns through classical energy method [6] where bamboo columns were considered as non-prismatic tubular members with varying second moment of area and Young's modulus along the member length. However, this design procedure was considered to be very lengthy and tedious in some causes, and simple design rule was preferred for practical design.

2. Objectives and scope of work

In order to promote the effective use of structural bamboo in building construction, a research and development program was undertaken by the second and the third authors from 1999–2001. The program aims to generate scientific design rules and data for the re-engineering of bamboo scaffolds into modern green structures of high buildability through scientific investigation and technology transfer. It is essential to provide not only design data of both physical and mechanical properties but also design rules against various modes of failure in accordance with modern design philosophy. After establishing the characteristic values of both the strengths and the Young's moduli of Kao Jue and Mao Jue [9], it is necessary to develop design rules against column buckling of bamboo culms.

A research and development project was carried out by the authors from 2000–2001 and the column buckling behaviour of structural bamboo was examined. A total of 72 column buckling tests for both Kao Jue and Mao Jue over a wide range of practical member lengths were executed to examine their column buckling behaviour. In accordance with existing structural design philosophy on column buckling for both steel and timber structures, a design method based on modified slenderness was then proposed for general design of both Kao Jue and Mao Jue after careful calibration against test data. It should

Table 1
Proposed mechanical properties for structural bamboo

Bamboo species	Compression			Bending		
		Dry	Wet		Dry	Wet
Bambusa pervariabilis (Kao Jue)						
Characteristic strength (N/mm^2) (at fifth percentile)	$f_{c,k}$	79	35		80	37
Design strength (N/mm^2) ($\gamma_m = 1.5$)	$f_{c,d}$	53	23	$f_{b,d}$	53	25
Design Young's modulus (kN/mm^2) (average value)	$E_{c,d}$	10.3	6.8	$E_{b,d}$	22.0	16.4
Phyllostachys pubescens (Mao Jue)						
Characteristic strength (N/mm^2) (at fifth percentile)	$f_{c,k}$	117	44	$f_{b,k}$	51	55
Design strength (N/mm^2) ($\gamma_m = 1.5$)	$f_{c,d}$	78	29	$f_{b,d}$	34	37
Design Young's modulus (kN/mm^2) (average value)	$E_{c,d}$	9.4	6.4	$E_{b,d}$	13.2	9.6

Dry condition: m.c. $<5\%$ for both Kao Jue and Mao Jue. Wet condition: m.c. $>20\%$ for Kao Jue and m.c. $>30\%$ for Mao Jue. Linear interpolation is permitted for mechanical properties with moisture contents between dry and wet conditions. The shear strengths of both Kao Jue and Mao Jue are conservatively estimated as 0.25 $f_{c,d}$, but not less than 6 N/mm^2 and greater than 15 N/mm^2.

be noted that any significant variation on the physical and the mechanical properties along the member length of bamboo columns should be incorporated in assessing their axial buckling resistances. Other aspects of the design and construction of bamboo scaffolds such as member configurations, support arrangement, and connection design will be reported separately.

3. Experimental investigation

In order to examine the buckling behaviour of bamboo culms and to provide test data for the formulation of design rules against column buckling, a set of systematic test series, which is also referred as a qualification test programme, for each bamboo species was executed. In each test programme, a large number of column buckling tests were carried out over a practical range of height-to-diameter ratios, diameter variations over member length, and also moisture content. In general, the test specimens were selected and prepared as follows:

- All bamboo culms were about 6 m in length and of 3 to 6 years of age. They were air-dried for at least 3 months before testing.
- A length of 750 mm from both the top and the bottom ends of the bamboo culms was discarded.
- Three specimens were cut out from the top, the middle and the bottom positions of the culm and marked with the letters *A*, *B*, and *C* respectively.
- All the specimens were fairly straight with acceptable out-of-straightness under visual inspection. The external diameters at the top and the bottom ends of each specimen did not differ by more than 25 mm.
- Three member lengths were selected, and they were 400, 600 and 800 mm for Kao Jue and 1000, 1500 and 2000 mm for Mao Jue; the member lengths were denoted as *a*, *b* and *c* respectively.

Among all the physical properties, moisture content was found to be the most important one in governing the mechanical properties, and hence, the buckling behaviour of bamboo culms. Consequently, it is necessary to perform the systematic tests with test specimens under the following moisture conditions:

- The natural condition is denoted as *N* and it represents the typical range of moisture contents found in practice.
- The wet condition is denoted as *W* and it represents the extreme moisture content approaching water saturation in bamboo fibers; this is achieved by immersing the test specimens under water for 1 week before testing.

The designation system for the test specimens are defined as follows:

$$\begin{Bmatrix} K \\ M \end{Bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} \begin{Bmatrix} N \\ W \end{Bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

where *K* and *M* denote Kao Jue and Mao Jue, respectively. The total number of tests is equal to 2×3×2×3×2 or 72.

It should be noted that after each column buckling test, at least two short culms were cut out from the buckled specimens, and compression tests on the short culms were carried out to evaluate the compressive strengths of the bamboo culms. The length of each compression test specimen was about twice the external diameter of the bamboo culm, but not larger than 75 and 150 mm for Kao Jue and Mao Jue, respectively.

3.1. Test set-up

Fig. 4 illustrated the general set-up of the column buckling tests. In order to simplify data analysis, smooth ball joints were installed to provide simply supported conditions at the top and the bottom supports. Therefore, the effective length coefficient of all the test specimens were taken as 1.0 in analysis. The applied load, *P*, the axial shortening, *w*, and the horizontal displacements, *u* and *v*, were measured continuously during the test to provide load-displacement curves for data analysis, and the maximum applied load and the corresponding displacements at failure were obtained for each test.

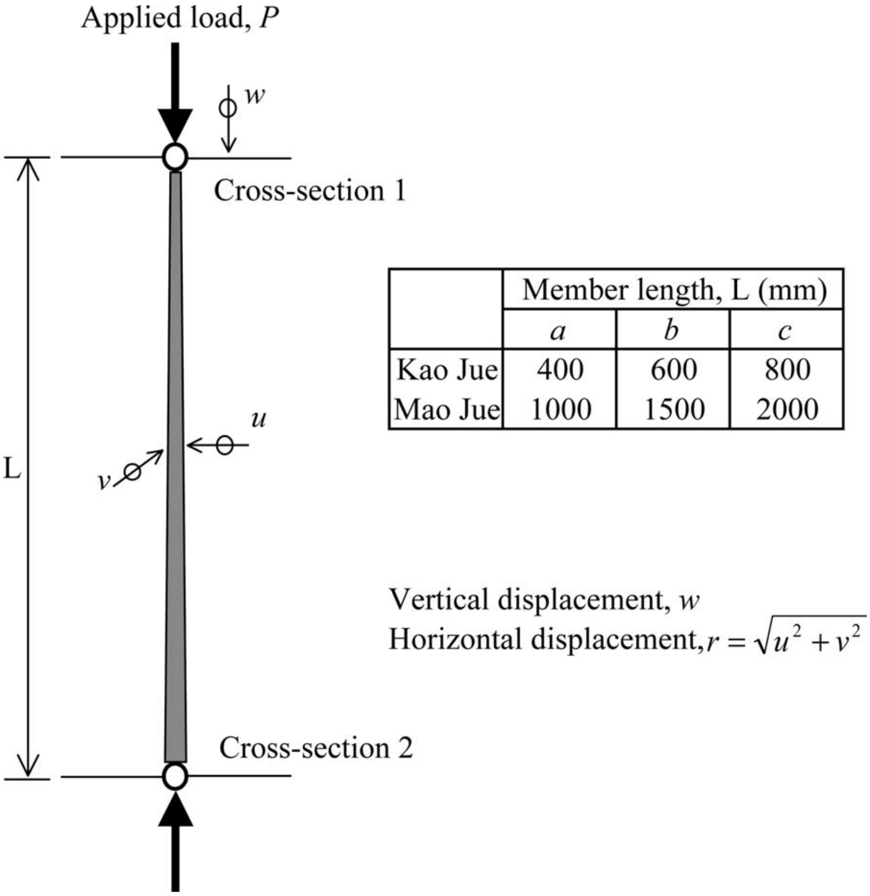
3.2. Test results

Two failure modes, namely *overall buckling* and *local buckling*, are identified among the tests, and they are shown in Fig. 5. It is found that most Mao Jue members fail in overall buckling, especially for those long columns with high moisture contents. For wet and short columns of Kao Jue, local buckling is critical.

All the measured data of the test specimens including geometrical dimensions, moisture contents and failure loads are presented in Tables 2 and 3 while typical load-displacement curves of the test specimens are plotted in Fig. 6. It is shown that the load reduction due to column buckling of the test specimens are severe, and thus it is necessary to derive a general design method to assess the axial buckling resistances of bamboo columns.

4. Design of bamboo column against buckling

Based on modern structural design philosophy, a design method is proposed for column buckling of both Kao Jue and Mao Jue in a limit state design format. The proposed design method follows closely to the column



	Member length, L (mm)		
	a	*b*	*c*
Kao Jue	400	600	800
Mao Jue	1000	1500	2000

Fig. 4. General set-up of column buckling tests.

buckling design methods of structural steel as given in the British steel code BS5950 [15]. It should be noted that the formulation of the proposed design method adopts the Perry-Robertson interaction formula to evaluate the compressive buckling strength of steel columns after incorporating the effects of both geometrical and material initial imperfections. In general, the values of both the Perry factor and the Robertson constant may be chosen in such a way to fit test data for column buckling of steel columns with different cross-sections buckling about different axes.

Moreover, the same method with a slightly different formulation is adopted in both the European Steel Code Eurocode 3 [16] and the European Timber Code Eurocode 5 [1]. Formulation of the non-dimensionalized column buckling curves with modified slenderness is presented in the Appendix together with a comparison on the expressions and the values of the Perry factor, the Robertson constant and the limiting slenderness among various design rules.

As natural non-homogenous organic materials, large variations of both physical and mechanical properties along the length of bamboo culms are apparent, and it is important to decide which parameters should be incorporated in assessing the axial buckling resistance of the bamboo columns. According to the results of the qualification test programs for Kao Jue and Mao Jue reported by Chung and Yu [9], the variation of Young's modulus along member length is found to be significant, and, more importantly, also random in pattern. Thus, the average value or 50 percentile of the Young's modulus should be adopted for the entire member length of both Kao Jue and Mao Jue.

However, the variations of external diameter and thickness in Mao Jue are apparent, and it is essential to allow for the variation of the second moment of area along the member length in the column buckling analysis. This may be readily achieved by incorporating a non-prismatic parameter, α, to the elastic Euler buckling load of bamboo columns. The non-prismatic parameter, α, may be evaluated through classical energy method. It is worthwhile to note that the International Standard ISO 22156 Bamboo Structural Design [17] recommends at least a 10% reduction to be applied to the second moment of area of bamboo culms whenever the cross-section variation along the member length is significant.

(a) Overall buckling (b) Local buckling

Fig. 5. Typical failure modes of bamboo columns. (a) Overall buckling; (b) Local buckling.

5. Proposed design method

The proposed design method for bamboo columns against buckling is presented as follows:

1. Basic section properties of a bamboo column are evaluated first:

   Cross − sectional area: $A_1 = \left[\frac{\pi}{4}(D_e^2 - D_i^2)\right]_1$

   Second moment of area: $I_1 = \left[\frac{\pi}{64}(D_e^4 - D_i^4)\right]_1; I_2 = \left[\frac{\pi}{64}(D_e^4 - D_i^4)\right]_2$

   Slenderness ratio: $\lambda_1 = \frac{L_E}{r_1}$ where $r_1 = \sqrt{\frac{I_1}{A_1}}$

   where subscripts 1 and 2 denote the upper (smaller) cross-section and the lower (larger) cross-section, respectively.
2. The elastic critical buckling strength of the bamboo column, f_{cr}, is given by:

   $$f_{cr} = \alpha \cdot \frac{\pi^2 E_{b,d}}{\lambda_1^2}$$

   where the non-prismatic parameter, α, is the minimum root of the following cubic function, $g(\alpha) = c_3\alpha^3 + c_2\alpha^2 + c_1\alpha + c_0 = 0$ where

   $$c_3 = -0.2880$$
   $$c_2 = 2.016\,(2 + \rho)$$
   $$c_1 = -(14.11 + 14.11\rho + 3.098\,\rho^2)$$
   $$c_0 = 10.37 + 15.55\rho + 7.047\rho^2 + 0.932\rho^3$$
   $$\rho = \frac{I_2 - I_1}{I_1}$$

   If the value of ρ lies between 0 and 3, the value of α may be evaluated approximately as follows:

   $$\alpha = 1.005 + 0.4751\rho - 0.011\rho^2$$

   where α lies between 1.00 and 2.35.
3. The design compressive strength of the bamboo column, $f_{c,d}$, is given by:

   $$f_{c,d} = \frac{f_{c,k}}{\gamma_m}$$
4. The design compressive buckling strength of the bamboo column, $f_{cc,d}$, is thus given by:

   $$f_{cc,d} = \frac{f_{cr}\, f_{c,d}}{\phi + (\phi^2 - f_{cr}\, f_{c,d})^{1/2}}$$

   where

   $$\phi = \frac{f_{c,d} + (1 + \eta)\, f_{cr}}{2}$$

Table 2
Details of test series for Kao Jue

Specimen	D_1 (mm)	d_1 (mm)	D_2 (mm)	d_2 (mm)	L (mm)	P_{test} (kN)	*m.c.* (%)	$f_{cc,d}$ (N/mm^2)	$f_{cc,t}$ (N/mm^2)	$\bar{\lambda}$	$\bar{\psi}_c$	$\bar{\psi}_t$	MF
KANa1	38.77	30.02	39.82	30.23	401	28.99	12.0	33.5	61.3	0.51	0.60	1.10	1.83
KBNa1	42.71	31.85	43.69	32.06	396	44.18	12.5	35.9	69.5	0.46	0.64	1.24	1.94
KCNa1	45.14	30.52	44.86	28.72	398	47.65	12.5	36.2	54.8	0.47	0.65	0.98	1.51
KANa2	41.02	33.23	42.48	34.42	399	33.44	11.6	35.5	73.6	0.47	0.63	1.31	2.08
KBNa2	45.08	35.45	45.51	35.04	401	34.70	11.9	37.4	57.0	0.44	0.67	1.02	1.53
KCNa2	45.14	33.69	44.08	31.44	398	28.42	12.5	37.0	40.1	0.45	0.66	0.72	1.08
KANb1	41.82	32.31	43.68	34.39	598	33.34	11.9	25.6	60.2	0.70	0.46	1.08	2.35
KBNb1	45.28	34.85	44.88	33.48	597	29.09	12.5	27.3	44.3	0.67	0.49	0.79	1.62
KCNb1	43.69	30.16	43.18	28.71	598	26.58	12.5	25.5	33.9	0.72	0.46	0.61	1.33
KANb2	36.38	27.91	37.33	27.68	598	13.04	11.5	22.4	30.5	0.80	0.40	0.54	1.36
KBNb2	40.11	30.22	40.08	28.92	600	18.94	11.5	24.2	34.7	0.75	0.43	0.62	1.43
KCNb2	40.84	27.37	41.29	24.44	600	20.68	12.5	23.9	28.7	0.76	0.43	0.51	1.20
KANc1	35.48	27.05	38.99	29.37	795	14.78	11.3	16.5	35.7	1.03	0.30	0.64	2.16
KBNc1	41.53	31.45	43.41	31.05	797	24.74	12.2	19.2	42.8	0.92	0.34	0.77	2.23
KCNc1	44.32	30.86	44.24	27.64	796	33.83	12.5	19.6	42.6	0.92	0.35	0.76	2.17
KANc2	33.56	26.58	36.60	29.52	799	5.80	11.6	15.4	17.6	1.11	0.28	0.31	1.14
KBNc2	38.57	30.92	41.39	33.29	798	9.47	10.8	18.1	22.7	0.97	0.32	0.41	1.25
KCNc2	42.91	35.39	44.00	34.37	799	11.02	11.3	20.5	23.8	0.86	0.37	0.43	1.16
KAWa1	36.42	28.45	37.52	29.13	397	18.75	49.5	23.7	46.2	0.53	0.58	1.13	1.95
KBWa1	41.87	32.20	42.11	31.55	401	22.23	51.8	25.9	39.5	0.47	0.63	0.96	1.52
KCWa1	44.12	31.60	44.73	31.05	399	29.87	43.7	26.5	40.1	0.46	0.65	0.98	1.51
KAWa2	39.21	31.31	40.40	32.60	394	21.55	55.3	25.3	49.3	0.49	0.62	1.20	1.95
KBWa2	43.90	35.29	44.48	35.25	396	24.55	53.4	27.3	45.8	0.44	0.67	1.12	1.68
KCWa2	45.01	34.69	44.32	33.56	400	17.78	54.6	27.3	27.5	0.45	0.67	0.67	1.01
KAWb1	38.92	30.38	42.29	33.23	597	20.01	55.9	17.9	43.0	0.71	0.44	1.05	2.40
KBWb1	44.12	34.41	45.25	34.20	597	25.13	84.0	19.9	42.0	0.65	0.49	1.02	2.11
KCWb1	45.45	34.44	45.17	31.35	598	16.92	54.4	20.1	24.5	0.65	0.49	0.60	1.22
KAWb2	33.69	25.98	35.16	26.18	596	10.14	70.4	15.4	28.1	0.84	0.38	0.68	1.82
KBWb2	38.56	29.56	39.68	28.09	598	15.08	67.2	17.6	31.3	0.73	0.43	0.76	1.78
KCWb2	40.24	28.93	41.05	28.45	598	20.10	57.5	17.7	32.7	0.74	0.43	0.80	1.85
KAWc1	34.95	27.66	39.10	30.86	801	9.47	79.2	12.1	26.4	1.01	0.30	0.64	2.18
KBWc1	41.26	32.59	42.78	33.25	800	9.95	81.3	14.1	19.8	0.92	0.34	0.48	1.41
KCWc1	42.40	31.34	42.66	31.27	799	9.47	84.0	13.9	14.8	0.96	0.34	0.36	1.06
KAWc2	36.28	26.96	38.08	28.18	802	14.40	77.6	11.9	31.1	1.07	0.29	0.76	2.62
KBWc2	40.54	30.48	41.95	31.27	796	20.98	72.6	13.6	37.4	0.96	0.33	0.91	2.76
KCWc2	41.08	29.68	41.52	26.30	804	22.43	64.5	13.5	35.4	0.96	0.33	0.86	2.63

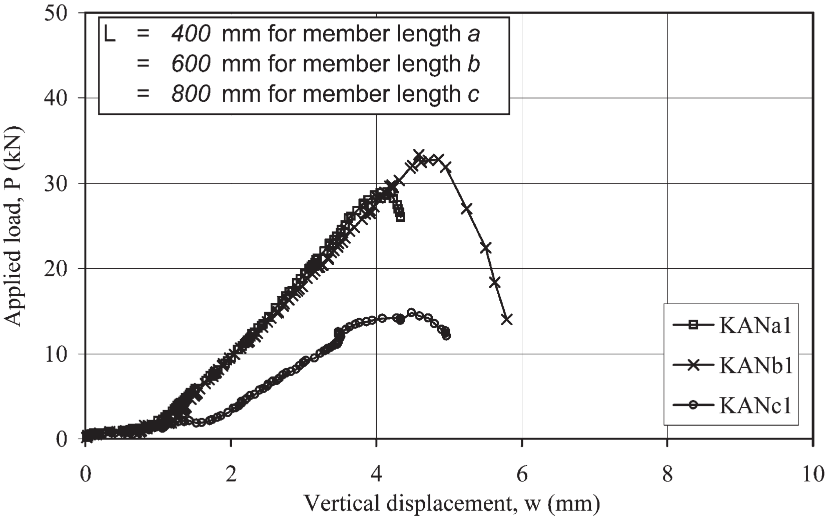
$f_{c,k}/\gamma_m = 56$ N/mm^2 with $\gamma_m = 1.0$ for natural condition. $f_{c,k}/\gamma_m = 41$ N/mm^2 with $\gamma_m = 1.0$ for wet condition. MF denotes the model factor $\bar{\psi}_t$ / $\bar{\psi}_c$. The average model factors for both natural and wet conditions are 1.63 and 1.86, respectively.

Table 3
Details of test series for Mao Jue

Specimen	D_1 (mm)	d_1 (mm)	D_2 (mm)	d_2 (mm)	L (mm)	P_{test} (kN)	*m.c* (%)	$f_{cc,d}$ (N/mm^2)	$f_{cc,t}$ (N/mm^2)	$\bar{\lambda}$	$\bar{\psi}_c$	$\bar{\psi}_t$	MF
MANa1	46.04	35.81	52.57	40.58	1000	27.64	14.2	17.5	42.0	1.11	0.38	0.91	2.41
MBNa1	76.65	62.29	81.96	66.64	1004	60.19	14.3	27.9	38.4	0.71	0.61	0.84	1.38
MCNa1	85.97	69.80	87.46	70.87	999	64.28	14.8	29.9	32.5	0.67	0.65	0.71	1.09
MANa2	77.84	64.50	83.71	69.09	1000	43.59	14.4	28.5	29.2	0.69	0.62	0.64	1.03
MBNa2	60.61	48.30	67.14	53.74	1001	37.60	14.3	23.0	35.7	0.87	0.50	0.78	1.55
MCNa2	83.23	67.85	86.76	69.53	1002	54.81	12.0	29.5	30.0	0.66	0.64	0.65	1.02
MANb1	63.76	52.35	73.98	60.83	1499	27.06	13.8	16.4	26.0	1.16	0.36	0.57	1.58
MBNb1	77.24	63.98	90.47	75.74	1500	43.11	14.4	20.5	29.3	0.96	0.45	0.64	1.43
MCNb1	95.19	77.24	103.05	83.10	1498	60.22	23.8	24.1	24.8	0.84	0.52	0.54	1.03
MANb2	51.27	40.03	60.75	47.10	1498	18.08	14.0	12.4	22.4	1.43	0.27	0.49	1.81
MBNb2	69.20	54.25	75.02	58.28	1497	42.43	18.0	16.9	29.3	1.17	0.37	0.64	1.74
MCNb2	68.25	53.63	77.92	59.07	1501	39.53	15.7	17.5	28.2	1.10	0.38	0.61	1.61
MANc1	59.75	48.55	71.23	57.52	2000	15.08	13.8	10.5	15.8	1.60	0.23	0.34	1.50
MBNc1	77.53	62.72	85.99	68.37	2001	41.76	16.4	14.0	25.6	1.33	0.31	0.56	1.82
MCNc1	73.22	58.04	86.45	68.43	2002	46.01	15.0	13.6	29.4	1.34	0.30	0.64	2.17
MANc2	54.82	45.32	67.91	54.62	1997	9.38	14.7	10.1	12.6	1.61	0.22	0.27	1.24
MBNc2	75.89	61.75	88.48	71.94	1999	25.03	13.9	14.3	16.4	1.30	0.31	0.36	1.15
MCNc2	89.22	73.58	99.27	80.16	1999	37.31	13.9	17.1	18.7	1.14	0.37	0.41	1.09
MAWa1	59.38	50.52	64.91	55.62	998	23.19	28.0	20.1	30.3	1.01	0.46	0.69	1.51
MBWa1	74.03	60.31	81.62	66.23	998	48.53	33.1	24.8	33.5	0.81	0.56	0.76	1.35
MCWa1	85.56	68.51	94.09	74.91	1001	79.07	24.0	27.6	38.3	0.71	0.63	0.87	1.39
MAWa2	54.16	42.16	62.11	47.27	998	28.32	32.4	18.4	31.2	1.06	0.42	0.71	1.70
MBWa2	65.18	52.21	71.50	56.69	998	42.33	32.1	21.9	35.4	0.93	0.50	0.81	1.62
MCWa2	88.32	71.72	94.64	76.57	997	75.68	20.5	28.1	36.3	0.70	0.64	0.82	1.29
MAWb1	57.86	46.20	67.39	55.12	1499	23.19	20.8	11.8	24.3	1.51	0.27	0.55	2.07
MBWb1	70.90	58.54	78.95	65.16	1498	34.79	27.6	15.4	27.7	1.25	0.35	0.63	1.80
MCWb1	75.30	60.45	86.69	67.51	1501	64.86	22.0	17.2	41.0	1.12	0.39	0.93	2.39
MAWb2	70.27	56.29	77.37	62.13	1499	33.93	34.0	14.8	24.4	1.30	0.34	0.56	1.65
MBWb2	60.18	47.41	69.54	54.70	1498	22.71	58.7	12.5	21.0	1.44	0.29	0.48	1.68
MCWb2	75.80	61.09	82.79	64.79	1500	35.67	38.8	16.5	22.6	1.18	0.38	0.51	1.36
MAWc1	42.64	32.47	58.92	45.95	1999	10.14	18.5	6.0	16.9	2.22	0.14	0.38	2.82
MBWc1	72.68	59.00	84.76	67.81	1998	21.46	27.9	11.3	15.2	1.53	0.26	0.35	1.34
MCWc1	69.26	55.56	83.38	66.67	2000	25.84	20.1	10.8	19.2	1.57	0.25	0.44	1.78
MAWc2	53.31	42.73	68.09	54.42	1998	8.12	22.8	7.9	10.2	1.89	0.18	0.23	1.29
MBWc2	63.85	50.01	78.68	62.40	1999	19.62	22.4	9.6	15.9	1.69	0.22	0.36	1.65
MCWc2	79.73	63.24	92.90	71.71	2001	31.61	22.5	12.8	17.1	1.41	0.29	0.39	1.34

$f_{c,k}/\gamma_m = 46$ N/mm^2 with $\gamma_m = 1.0$ for natural condition. $f_{c,k}/\gamma_m = 44$ N/mm^2 with $\gamma_m = 1.0$ for wet condition. MF denotes the model factor $\overline{\psi_t} / \overline{\psi_c}$. The average model factors for natural and wet conditions are 1.48 and 1.67, respectively.

a) Kao Jue



b) Mao Jue

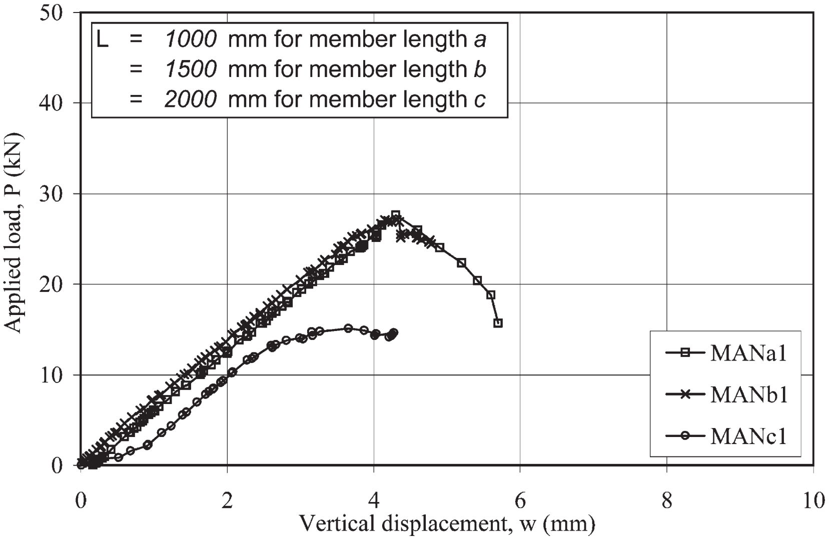


Fig. 6. Typical load displacement curves of column buckling tests. (a) Kao Jue; (b) Mao Jue.

Perry factor, $\eta = 0.001\, a\, (\lambda_1 - \lambda_0)$

Robertson constant, $a = 15$ for Mao Jue, or

$a = 28$ for Kao Jue.

Limiting slenderness, $\lambda_o = 0.2\ \pi \sqrt{\dfrac{E_{b,d}}{f_{c,d}}}$

A non-dimensionalized column buckling curve may be plotted using the following two non-dimensionalized quantities:

- Modified slenderness ratio, $\bar{\lambda} = \sqrt{\dfrac{f_{c,d}}{f_{cr}}}$
- Strength reduction factor, $\bar{\psi}_c = \dfrac{f_{cc,d}}{f_{c,d}}$

The compressive buckling strength of a bamboo column is thus obtained as a factor of the compressive strength. For illustration purpose, Figs. 7 and 8 plot the proposed column buckling curves for both Kao Jue and Mao Jue, respectively.

5.1. Calibration of design method

In order to calibrate the proposed design method, a back analysis against the test data was carried out with all partial safety factors equal to unity. Moreover, the measured dimensions of the test specimens were used, and the measured compressive strengths of test specimens under natural and wet conditions were adopted. It should be noted that the design values of Young's moduli against bending of both Kao Jue and Mao Jue as given in Table 1 were used in the back analysis.

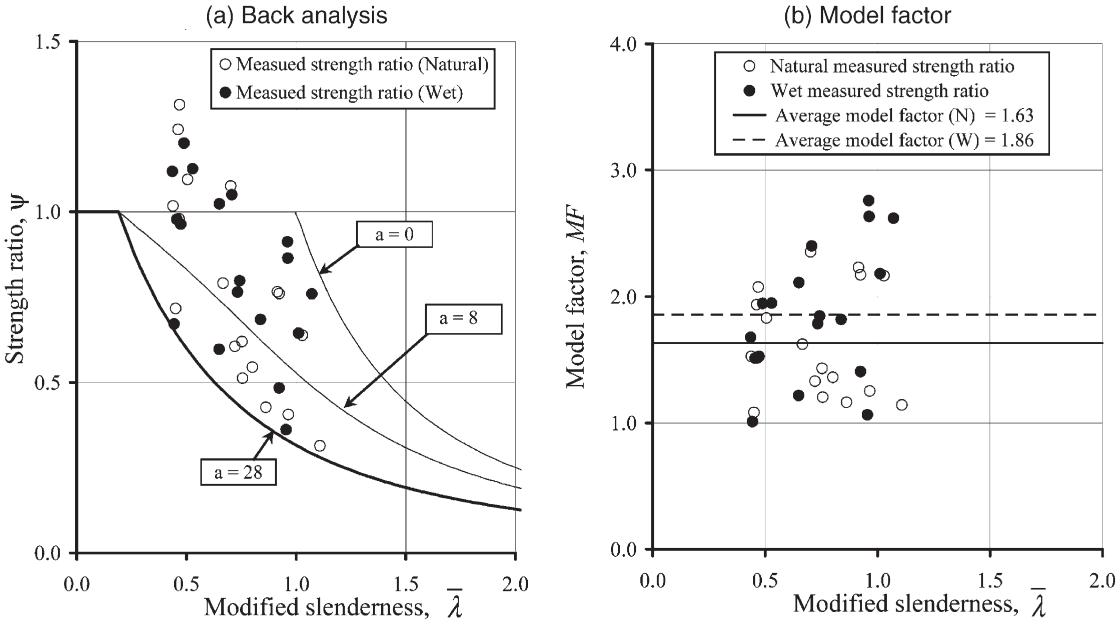


Fig. 7. Column buckling analysis for Kao Jue. (a) Back analysis; (b) Model factor.

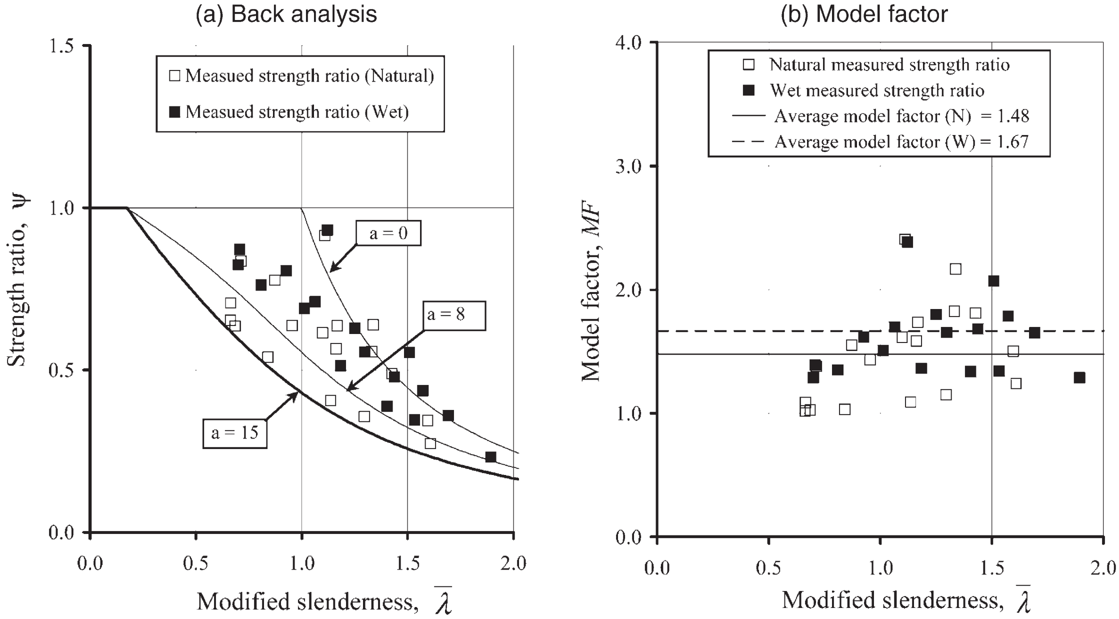


Fig. 8. Column buckling analysis for Mao Jue.

The results of the back analysis for both Kao Jue and Mao Jue are summarized in Tables 2 and 3 while the test data is also plotted in Figs. 7 and 8 respectively for direct comparison with the proposed column buckling curves. It should be noted that:

- For Kao Jue, it is found that due to the presence of large initial imperfection when compared with the external diameter, the Robertson constant is selected to be 28 in order to give safe design for all test results. The measured modified slenderness ratios are found to range from 0.44–1.11, while the measured strength reduction ratios are found to range from 0.31–1.31.
- For Mao Jue, the Robertson constant is selected to be 15 due to small initial imperfection when compared with the external diameter. The measured modified slenderness ratios are found to range from 0.66–2.22 and the measured strength reduction ratios are found to range from 0.23–0.93.

The model factors for the proposed design method of column buckling against the test data of both Kao Jue and Mao Jue are also presented in Tables 2 and 3, respectively. The distributions of the model factors for both Kao Jue and Mao Jue under different moisture conditions are plotted in Figs. 7 and 8, respectively. For Kao Jue, the average model factors are found to be 1.63 and 1.86 for natural and wet conditions, respectively. Similarly, the average model factors for Mao Jue are found to be 1.48 and 1.67 for natural and wet conditions, respectively. Consequently, the proposed design method is shown to be adequate.

It should be noted that in the present study, the value of non-prismatic parameter, α, for Kao Jue is found to range from 1.00–1.28, and thus, the variation of external diameter and thickness in bamboo columns of Kao Jue is considered not to be significant in assessing their axial buckling resistances. However, for Mao Jue, α is found to range from 1.04–2.11, and thus, it is essential to incorporate the variation of external diameter and thickness, and hence, second moment of area, in bamboo columns of Mao Jue in assessing their axial buckling resistance.

6. Practical considerations

Attention should be drawn to the following for practical design and construction of bamboo scaffolds and structures.

6.1. Basic design data for Kao Jue and Mao Jue

It is important to have statistically corrected engineering data for structural bamboo so that rational design of bamboo structures is possible. For general application, the following data may be adopted:

- For Kao Jue, the external and the internal diameters are 40 and 30 mm, respectively, and they are constant along the length of the bamboo culm; the wall thickness is 5 mm.
- For Mao Jue, the external and the internal diameters at the top cross-section are 60 and 48 mm, respectively, and they are considered to increase linearly down to the bottom cross-section to 90 and 72 mm, respectively, over a length of 6 m. The wall thickness increases linearly from 6 mm at the top cross-section to 9 mm at the bottom cross-section.

The dimensions of Kao Jue and Mao Jue are illustrated in Fig. 3 while the design data of the mechanical properties for both Kao Jue and Mao Jue is presented in Table 1. All bamboo culms should be air-dried for three months before use and free from visual defects.

6.2. Initial out-of-straightness

As non-homogenous organic materials, most bamboo culms are found to have initial out-of straightness with different magnitudes among individual members. It is important to limit the maximum initial out-of-straightness, $\Delta_{o,max}$, found in practice in order to ensure that the proposed design method against column buckling for structural bamboo is valid. Table 4 presents the average values of the initial out-of-straightness measured from the column buckling tests. As a rule of thumb, the maximum value of initial out-of-straightness, $\Delta_{o,max}$, should be limited as follows:

For Kao Jue: $\Delta_{o,max} = L/100$ or 0.15 D_e, whichever is smaller.

For Mao Jue: $\Delta_{o,max} = L/200$ or 0.15 D_e, whichever is smaller.

6.3. Lateral restraints in bamboo scaffolds

In order to achieve overall structural stability of bamboo scaffolds with high structural efficiency, lateral restraints should be provided at close intervals. However, in practise, it may not be practical or even impossible to provide lateral restraints in all the main post-main ledger connections of the bamboo scaffolds. In the absence of sufficient lateral restraints, the effective length of bamboo columns will be larger than their member lengths between ledgers, and the axial buckling resistance of the bamboo columns may be reduced significantly. Thus, it is necessary to provide design guidance on practical arrangements of lateral restraints.

An advanced non-linear finite element analysis has been carried out [2] to investigate the column buckling behaviour of bamboo scaffolds based on high performance beam-column elements using the one-element-per-member formulation [18]. Both the local buckling of bamboo posts between ledgers and the global instability of the entire scaffolds with regular and staggered lateral restraints were studied carefully. Details of the investigation may be found in the literature [2].

Table 4
Summary of initial out-of-straightness for structural bamboo

Bamboo species	Member length	Average out-of-straightness, Δ_0 (mm)	Ratios on initial out-of-straightness	
			Δ_o / L	Δ_o / D_e
Kao Jue				
a	400	3.1	L / 130	0.08
b	600	5.9	L / 102	0.15
c	800	3.0	L / 264	0.08
Maximum value			L / 102	0.15
Mao Jue				
a	1000	1.7	L / 595	0.02
b	1500	3.1	L / 492	0.04
c	2000	8.8	L / 226	0.13
Maximum value			L / 226	0.13

D_e = 40 mm for Kao Jue, 70 mm for Mao Jue.

7. Conclusions

Based on extensive and systematic experimental testing on the column buckling behaviour of bamboo culms, a design method against column buckling of structural bamboo based on modified slenderness is developed and calibrated successfully against test data of both Kao Jue and Mao Jue. For design purpose, both Kao Jue and Mao Jue may be considered to be homogenous in terms of mechanical properties. Moreover, the cross section dimensions of Kao Jue may be considered to be uniform throughout the length of bamboo culms against column buckling. However, the variations of the external diameter and the thickness, and hence, the second moment of area, in Mao Jue should be incorporated in assessing the axial buckling resistances of bamboo culms.

The proposed design method is shown to be structurally adequate in accordance with modern structural design philosophy, and it may be used effectively to design against column buckling of structural bamboo in bamboo scaffolds and other bamboo structures. With the availability of design data on the dimensions and the mechanical properties of structural bamboo together with the proposed column buckling design rule, structural engineers are encouraged to take the advantage offered by bamboo to build light and strong bamboo structures to achieve enhanced economy and buildability.

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Appendix. Formulation of non-dimensionalized column buckling curve

From classical energy method, the critical buckling load of a column is given by:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Dividing by the cross-sectional area, A, the elastic critical buckling strength, f_{cr}, is given by:

$$f_{cr} = \pi^2 E \frac{I}{A} \frac{1}{L^2}$$

$$f_{cr} = \pi^2 E \frac{1}{(L/r)^2} \text{ where } \frac{I}{A} = r^2 \text{ or } f_{cr} = \pi^2 E \frac{1}{\lambda^2} \text{ where } \lambda = \frac{L}{r}$$

Adopt the following Perry–Robertson interaction formula:

$$(f_{cr} - f_{cc,d})(f_{c,d} - f_{cc,d}) = \eta\, f_{cr} f_{cc,d}$$

The compressive buckling strength, $f_{cc,d}$, of the column is given by:

$$f_{cc,d} = \frac{f_{cr} f_{c,d}}{\phi + \sqrt{\phi^2 - f_{cr} f_{c,d}}}$$

where $\phi = \frac{f_{c,d} + (1+\eta) f_{cr}}{2}$

η = Perry factor which is related to initial imperfection of column member $= 0.001\, a(\lambda - \lambda_0)$

$f_{c,d}$ = design compressive strength

This is the same expression given in Appendix C of BS5950: Part 1. For both Eurocodes 3 and 5, a non-dimensionalized column buckling curve is adopted through the use of the following two non-dimensionalized quantities:

- Modified slenderness ratio, $\bar{\lambda}$ is defined by:

  $$\bar{\lambda} = \sqrt{\frac{f_{c,d}}{f_{cr}}} \text{ or } = \lambda / \lambda_1$$

  where λ is the slenderness of the column and

  $$\lambda_1 = \pi \sqrt{\frac{E}{f_{c,d}}}$$

- Reduction factor for axial buckling, χ is defined by:

  $$\chi = \frac{f_{cc,d}}{f_{c,d}} = \frac{f_{cr}}{\phi + (\phi^2 - f_{cr}\, f_{c,d})^{1/2}} = \frac{1}{\frac{\phi}{f_{cr}} + \sqrt{\left(\frac{\phi}{f_{cr}}\right)^2 - \frac{f_{c,d}}{f_{cr}}}}$$

  Re-writing the formula using different parameters,

  $$\chi = \frac{1}{\bar{\phi} + \sqrt{\bar{\phi}^2 - \bar{\lambda}^2}}$$

  where $\bar{\phi} = 0.5[\ 1 + \bar{a}\,(\bar{\lambda} - 0.2) + \bar{\lambda}^2\ ]$; $\bar{a}$, is an imperfection factor whose value depends on the material of the columns, and also the initial out-of-straightness of the column member allowed for. The value is given explicitly in Table 5.5.1 of EC3, or $0.001\ a\ \pi \sqrt{\frac{E}{f_{c,d}}}$.

The expression is similar to the design rule given in Clause 5.5.1.2 of ENV 1993-1-1:1992 of Eurocode 3. In clause 5.2.1 of ENV 1995-1-1:1993 of Eurocode 5, the same formulation is adopted but presented with different symbols as follows:

$$k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel}^2}}$$

where k_c, reduction factor due to column buckling;

k is $0.5\,[\,1 + \beta_c\,(\lambda_{rel} - 0.5) + \lambda_{rel}^2]$;

λ_{rel} is $\sqrt{\dfrac{f_{c,o,k}}{\sigma_{c,crit}}} = \sqrt{\dfrac{f_{c,d}}{f_{cr}}} = \bar{\lambda}$

and $\beta_c = \bar{a} = 0.001\ a\ \pi\sqrt{\dfrac{E}{f_{c,d}}}$

Summary of design rules

Approach of formulation	Perry factor	Robertson constant	Limiting slenderness (ratio)
British Steel Code BS5950 *Buckling strength*	$\eta = 0.001\ a\ (\lambda - \lambda_0)$	a = 2.0 for curve a; = 3.5 for curve b; = 5.5 for curve c; = 8.0 for curve d	$\lambda_0 = 0.2\ \pi\sqrt{\dfrac{E}{f_{c,d}}}$
European Steel Code EC3 *Modified slenderness*	$\eta = \bar{a}\ (\bar{\lambda} - 0.2)$	$\bar{a}$ = 0.21 for curve a; = 0.34 for curve b; = 0.49 for curve c; = 0.76 for curve d	$\lambda_0 = 0.2$
European Timber Code EC5 *Modified slenderness*	$\eta = \beta_c(\bar{\lambda} - 0.5)$	β_c = 0.1 for glued laminated timber; = 0.2 for solid timber	$\lambda_0 = 0.5$

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